

Graviton and scalar propagations on AdS_4 space in $f(R)$ gravities

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Abstract

We investigate propagations of graviton and additional scalar on four-dimensional anti de Sitter (AdS_4) space using $f(R)$ gravity models with external sources. It is shown that there is the van Dam-Veltman-Zakharov (vDVZ) discontinuity in $f(R)$ gravity models because $f(R)$ gravity implies GR with additional scalar. This indicates a difference between general relativity and $f(R)$ gravity clearly.

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1 Introduction

There has been much interest in the massless limit of the massive graviton propagator [1, 2, 3, 4, 5]. A key issue of this approach is that van Dam-Veltman-Zakharov (vDVZ) discontinuity [6] is peculiar to Minkowski space, but it does not arise in (anti) de Sitter space. The vDVZ discontinuity implies that the limit of $M^2 \rightarrow 0$ does not yield a massless graviton at the tree level such that the Einstein gravity (general relativity: GR) is isolated from the massive gravity. One has usually introduced the spin-2 Fierz-Pauli mass term with mass squared M_{FP}^2 [7] for this calculation. If the cosmological constant (CC, Λ) was introduced, the limit of $M_{\text{FP}}^2/\Lambda \rightarrow 0$ may recover a massless graviton. Another resolution to the discontinuity is possible to occur even in Minkowski space, if the Schwarzschild radius of the scattering objects is taken to be the second mass scale [8]. However, these all belong to the linearized (tree) level calculations. If one-loop graviton vacuum amplitude is considered for a massive graviton [9], the discontinuity appears again. This means that the apparent absence of the vDVZ discontinuity may be considered as an artifact of the tree level approximation. Also, there is the Boulware-Deser instability which states that at the non-linearized level, a ghost appears in the massive gravity theory [10].

On the other hand, $f(R)$ gravities as modified gravity theories [11, 12, 13] have much attentions as one of strong candidates for explaining the current accelerating universe [14]. Actually, $f(R)$ gravities can be considered as GR (massless graviton) with an additional scalar field. Explicitly, it was shown that the metric- $f(R)$ gravity is equivalent to the $\omega_{\text{BD}} = 0$ Brans-Dicke (BD) theory with the potential, while the Palatini- $f(R)$ gravity is equivalent to the $\omega_{\text{BD}} = -3/2$ BD theory with the potential [15]. However, it was pointed out that the mapping seems to be problematic because the potential defined by $U(\Phi(R)) = R\Phi - f(R)$ with $\Phi = \partial_R f(R)$ induces a singularity in the cosmological evolution [16, 17, 18]. Although the equivalence principle test (EPT) in the solar system imposes a strong constraint on $f(R)$ gravities, they may not be automatically ruled out if the Chameleon mechanism is introduced to resolve it. It was shown that the EPT allows $f(R)$ gravity models that are indistinguishable from the Λ CDM model (R +positive CC) in the background universe evolution [19]. However, this does not necessarily imply that there is no difference in the dynamics of perturbations [20].

There were perturbation studies for the propagation of graviton on the constant curvature background using a single $f(R)$ gravity [21], but the analysis is not complete because they did not calculate one-particle scattering amplitude with external sources $T_{\mu\nu}$. Also, it was argued that there is no vDVZ discontinuity in GR with higher curvature terms (for example, $R - 2\Lambda + \alpha R^2$) on AdS_4 space [22]. Recently, a similar analysis was performed

in D -dimensional anti-de Sitter (AdS_D) space, including the new massive gravity in three dimensions [23].

In this work, we investigate propagations of graviton and additional scalar on AdS₄ space using $f(R)$ gravity models with external sources. Furthermore, we show that the vDVZ discontinuity appears in $f(R)$ gravity models because $f(R)$ gravity means GR with an additional scalar.

2 $f(R)$ gravities

We start from $f(R)$ gravity without any matter including a cosmological constant

$$I = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \{R + f(R)\}. \quad (1)$$

This splitting form of “ $R + f(R)$ ” is rather treatable than a single “ $f(R)$ ” form. In this work, $f(R)$ gravity means the former form and we set $16\pi G = 2$. Also we follow the signature of $(-+++)$. The equation of motion is given by

$$R_{\mu\nu} [1 + f'(R)] - \frac{1}{2} g_{\mu\nu} [R + f(R)] + [g_{\mu\nu} \nabla^2 - \nabla_\mu \nabla_\nu] f'(R) = 0. \quad (2)$$

For the case of $f(R) = -2\Lambda$ ($f' = f'' = 0$) (equivalently, GR with CC), we have the Einstein-Hilbert action with a negative CC. In this case, the vacuum solution is the four dimensional anti de Sitter (AdS₄) space whose geometry is expressed in terms of the metric ($\bar{g}_{\mu\nu}$) as

$$\bar{R}_{\mu\nu\rho\sigma} = \frac{\Lambda}{3} (\bar{g}_{\mu\rho} \bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma} \bar{g}_{\nu\rho}), \quad \bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu}, \quad \bar{R} = 4\Lambda = -\frac{12}{\ell^2}. \quad (3)$$

Its line element takes the form

$$ds_{\text{AdS}}^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -\left(1 + \frac{r^2}{\ell^2}\right) dt^2 + \frac{dr^2}{\left(1 + \frac{r^2}{\ell^2}\right)} + r^2 d\Omega_2^2. \quad (4)$$

In order to find a similar AdS₄ space solution, one has to consider a constant curvature scalar $R = \bar{R}$ with $f'(\bar{R}) = \text{const}$ and $f''(\bar{R}) = \text{const}$. In this case, Eq.(2) leads to

$$\bar{R}_{\mu\nu} [1 + f'(\bar{R})] - \frac{1}{2} \bar{g}_{\mu\nu} [\bar{R} + f(\bar{R})] = 0 \quad (5)$$

which means that the third term in (2) plays no role in obtaining the constant curvature solution. However, it will play an important role in the perturbation analysis around

the background of AdS_4 space. Taking the trace of (5), \bar{R} is determined by an algebraic equation

$$[1 + f'(\bar{R})]\bar{R} - 2[\bar{R} + f(\bar{R})] = 0. \quad (6)$$

From this equation, one finds the constant curvature scalar as a function of $f(\bar{R})$ and $f'(\bar{R})$

$$\bar{R} = 2 \left[\frac{\bar{R} + f(\bar{R})}{1 + f'(\bar{R})} \right] = \frac{2f(\bar{R})}{f'(\bar{R}) - 1} \equiv 4\bar{\Lambda}_f, \quad (7)$$

where the last equivalence is established by analogy of (3). We call $\bar{\Lambda}_f$ an effective cosmological constant because it is not a genuine CC but it is an induced CC from $f(R)$ gravities. Similarly, from (5) we read off the Ricci tensor

$$\bar{R}_{\mu\nu} = \frac{1}{2} \left[\frac{\bar{R} + f(\bar{R})}{1 + f'(\bar{R})} \right] \bar{g}_{\mu\nu} = \bar{\Lambda}_f \bar{g}_{\mu\nu}. \quad (8)$$

Hence, as far as the AdS_4 vacuum solution is concerned, there is no essential difference between GR with CC ($R - 2\Lambda$) and $f(R)$ gravity. However, we have to distinguish two models by noting that $f'(\bar{R}) = \text{const}$ and $f''(\bar{R}) = \text{const}$ in $f(R)$ gravities. Furthermore, it is well known that metric $f(R)$ gravity (especially for $R + \alpha R^2$ [24]) is equivalent to the $\omega_{BD} = 0$ Brans-Dicke theory with the potential (scalar-tensor theory)¹. Therefore we expect from (1) that a massless graviton (2 degrees of freedom: 2DOF) and a massive scalar (1 DOF) propagate on AdS_4 space without any ghost.

3 Perturbation analysis

In order to study the propagation of the metric, we introduce the perturbation around the background of AdS_4 space

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \quad (9)$$

Hereafter we denote the background values with “overbar”. After a lengthy calculation, the linearized equation to Eq.(2) with the external source $T_{\mu\nu}$ takes the form

$$(1 + f'(\bar{R}))\delta G_{\mu\nu}(h) + f''(\bar{R})[\bar{g}_{\mu\nu}\bar{\nabla}^2 - \bar{\nabla}_\mu\bar{\nabla}_\nu + \bar{\Lambda}_f\bar{g}_{\mu\nu}]\delta R(h) = T_{\mu\nu}, \quad (10)$$

where the linearized Einstein tensor with an effective CC is given by [23]

$$\delta G_{\mu\nu}(h) = \delta R_{\mu\nu}(h) - \frac{\bar{g}_{\mu\nu}}{2}\delta R(h) - \bar{\Lambda}_f h_{\mu\nu}. \quad (11)$$

¹More explicitly, the metric $f(R)$ gravity is equivalent to the Brans-Dicke theory with the potential in the Jordan frame, while $f(R)$ gravities of $R + f(R)$ is equivalent to GR with a scalar field in the Einstein frame. Hence, the AdS_4 space solution is mapped into other constant curvature solution with specific solution for a scalar field.

The linearized Ricci tensor and the linearized scalar curvature take the forms, respectively,

$$\delta R_{\mu\nu}(h) = \frac{1}{2} \left[\bar{\nabla}^\rho \bar{\nabla}_\mu h_{\nu\rho} + \bar{\nabla}^\rho \bar{\nabla}_\nu h_{\mu\rho} - \bar{\nabla}^2 h_{\mu\nu} - \bar{\nabla}_\mu \bar{\nabla}_\nu h \right], \quad (12)$$

$$\delta R(h) = \bar{g}^{\mu\nu} \delta R_{\mu\nu}(h) - h^{\mu\nu} \bar{R}_{\mu\nu} = \bar{\nabla}^\rho \bar{\nabla}^\mu h_{\rho\mu} - \bar{\nabla}^2 h - \bar{\Lambda}_f h. \quad (13)$$

In deriving these, we used the Taylor expansions around the constant curvature scalar background $R = \bar{R}$ as

$$f(R) = f(\bar{R}) + f'(\bar{R})\delta R(h) + \dots, \quad (14)$$

$$f'(R) = f'(\bar{R}) + f''(\bar{R})\delta R(h) + \dots. \quad (15)$$

The trace of (10) has

$$\left[-(1 + f'(\bar{R})) + f''(\bar{R})(3\bar{\nabla}^2 + 4\bar{\Lambda}_f) \right] \delta R(h) = T. \quad (16)$$

At this stage, we note that the linearized equation (10) is invariant under linearized diffeomorphisms as

$$\delta_\xi h_{\mu\nu} = \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu, \quad (17)$$

because of

$$\delta_\xi \delta G_{\mu\nu}(h) = 0, \quad \delta_\xi \delta R(h) = 0. \quad (18)$$

This implies that divergence and double divergence do not provide any constraint on $h_{\mu\nu}$. Also, the gauge invariant (physical) quantity is still left undetermined by the linearized equation (10).

In order to find physically propagating modes, we decompose the metric perturbation $h_{\mu\nu}$ with 10 DOF into

$$h_{\mu\nu} = h_{\mu\nu}^{TT} + \bar{\nabla}_{(\mu} V_{\nu)} + \bar{\nabla}_\mu \bar{\nabla}_\nu \phi + \psi \bar{g}_{\mu\nu}, \quad (19)$$

where $h_{\mu\nu}^{TT}$ is the transverse traceless tensor with 5 DOF ($\bar{\nabla}^\mu h_{\mu\nu}^{TT} = 0, h^{TT} = 0$), V_ν is a divergence free vector with 3 DOF ($\bar{\nabla}^\mu V_\mu = 0$), and ϕ and ψ are scalar fields with 2 DOF. These imply two relations

$$\bar{\nabla}^2 h = \bar{\nabla}^4 \phi + 4\bar{\nabla}^2 \psi, \quad \bar{\nabla}^\mu \bar{\nabla}^\nu h_{\mu\nu} = \bar{\nabla}^4 \phi + \bar{\Lambda}_f \bar{\nabla}^2 \phi + \bar{\nabla}^2 \psi. \quad (20)$$

Up to now, we did not make any choice on the gauge-fixing. One-particle scattering amplitude is mostly computed by choosing a condition of

$$\bar{\nabla}^\mu \bar{\nabla}^\nu h_{\mu\nu} = \bar{\nabla}^2 h. \quad (21)$$

When the mass term is present, this condition could be derived naturally [22, 23, 25]. For example, adding $M_{\text{FP}}^2(h_{\mu\nu} - h\bar{g}_{\mu\nu})/2$ to the linearized equation (10) leads to the condition of $\bar{\nabla}^\mu h_{\mu\nu} = \bar{\nabla}_\nu h$ ((21)) when hitting single (double) divergence on it [23]. In $f(R)$ gravity theories, however, we do not consider any mass term. Hence, (21) could be obtained from a gauge-fixing of

$$\bar{\nabla}^\mu h_{\mu\nu} = \bar{\nabla}_\nu h. \quad (22)$$

Then, considering (20) together with this condition leads to

$$3\bar{\nabla}^2\psi = \bar{\Lambda}_f\bar{\nabla}^2\phi \quad (23)$$

which implies that two scalars ϕ and ψ are not independent under the condition of (21). Plugging this into the first relation of (20), one finds a relation between the trace of $h_{\mu\nu}$ and scalar ψ as

$$h = \left[\frac{3}{\bar{\Lambda}_f} \bar{\nabla}^2 + 4 \right] \psi. \quad (24)$$

Imposing (21) on (13) leads to $\delta R(h) = -\bar{\Lambda}_f h$. Using (16) and (24), we express ψ in terms of the trace T of external sources $T_{\mu\nu}$ as

$$\psi = \frac{1}{9f''(\bar{R}) \left[\frac{1+f'(\bar{R})}{3f''(\bar{R})} - (\bar{\nabla}^2 + \frac{4}{3}\bar{\Lambda}_f) \right] (\bar{\nabla}^2 + \frac{4}{3}\bar{\Lambda}_f)} T \quad (25)$$

which means that ψ becomes a massive scalar on AdS_4 space of $f(R)$ gravities.

In order to find the transverse traceless part $h_{\mu\nu}^{TT}$, we need the Lichnerowicz operator Δ_L acting on spin-2 symmetric tensors defined by

$$\Delta_L h_{\mu\nu} = -\bar{\nabla}^2 h_{\mu\nu} + \frac{8\bar{\Lambda}_f}{3} \left(h_{\mu\nu} - \frac{h}{4} \bar{g}_{\mu\nu} \right). \quad (26)$$

Taking into account this, we rewrite the linearized Einstein tensor as

$$\delta G_{\mu\nu}^{TT}(h) = \frac{\Delta_L h_{\mu\nu}^{TT}}{2} - \bar{\Lambda} h_{\mu\nu}^{TT}. \quad (27)$$

Hence, we express $h_{\mu\nu}^{TT}$ in terms of external sources as

$$h_{\mu\nu}^{TT} = \frac{2}{(1 + f'(\bar{R}))(\Delta_L - 2\bar{\Lambda}_f)} T_{\mu\nu}^{TT}, \quad (28)$$

where the transverse traceless source ($\nabla^\mu T_{\mu\nu}^{TT} = 0, T^{TT} = 0$) is given by [3]

$$T_{\mu\nu}^{TT} = T_{\mu\nu} - \frac{1}{3} T g_{\mu\nu} + \frac{1}{3} (\bar{\nabla}_\mu \bar{\nabla}_\nu + g_{\mu\nu} \bar{\Lambda}_f/3) (\bar{\nabla}^2 + 4\bar{\Lambda}_f/3)^{-1} T. \quad (29)$$

We are now in a position to define the tree-level (one particle) scattering amplitude between two external sources $\tilde{T}_{\mu\nu}$ and $T_{\mu\nu}$ as

$$A = \frac{1}{4} \int d^4x \sqrt{-\bar{g}} \tilde{T}_{\mu\nu}(x) h^{\mu\nu}(x) \equiv \frac{1}{4} [\tilde{T}_{\mu\nu} h^{TT\mu\nu} + \tilde{T}\psi], \quad (30)$$

where we suppress the integral to have a notational simplicity in the last expression. Finally, the scattering amplitude takes the form

$$\begin{aligned} 4A &= 2\tilde{T}_{\mu\nu} [(1 + f'(\bar{R}))(\Delta_L - 2\bar{\Lambda}_f)]^{-1} T^{\mu\nu} + \frac{2}{3} \tilde{T} [(1 + f'(\bar{R}))(\bar{\nabla}^2 + 2\bar{\Lambda}_f)]^{-1} T \\ &- \frac{2\bar{\Lambda}_f}{9} \tilde{T} [(1 + f'(\bar{R}))(\bar{\nabla}^2 + 2\bar{\Lambda}_f)]^{-1} \left[\bar{\nabla}^2 + \frac{4\bar{\Lambda}_f}{3} \right]^{-1} T \\ &+ \frac{1}{9f''(\bar{R})} \tilde{T} \left[\frac{1 + f'(\bar{R})}{3f''(\bar{R})} - \left(\bar{\nabla}^2 + \frac{4\bar{\Lambda}_f}{3} \right) \right]^{-1} \left[\bar{\nabla}^2 + \frac{4\bar{\Lambda}_f}{3} \right]^{-1} T. \end{aligned} \quad (31)$$

We note that as is shown in Eq.(7), the effective cosmological constant $\bar{\Lambda}_f$ is not an independent quantity but it is determined by $f'(\bar{R})$ and $f''(\bar{R})$.

4 van DVZ discontinuity

The expression of (31) is quite a nontrivial integral, but we can study the particle spectrum of graviton and scalar in $f(R)$ gravities by investigating the pole structure of the amplitude. We note that in $f(R)$ gravity, taking a limit of $\bar{\Lambda}_f \rightarrow 0$ is equivalent to the limit of $f(R) \rightarrow 0$, which is nothing but GR. Here we read off three poles from (31). We wish to compute the residue at each pole.

(a) Pole at $\bar{\nabla}^2 = -\frac{4\bar{\Lambda}_f}{3}$

The residue at this unphysical pole is zero as

$$-\frac{2\bar{\Lambda}_f}{9(1 + f'(\bar{R}))} \frac{1}{\left[-\frac{4\bar{\Lambda}_f}{3} + 2\bar{\Lambda}_f \right]} + \frac{1}{3(1 + f'(\bar{R}))} = 0. \quad (32)$$

(b) Pole at $\bar{\nabla}^2 = -2\bar{\Lambda}_f$

The residue at this physical pole takes the form

$$\frac{1}{1 + f'(\bar{R})} \left[\frac{2}{3} - \frac{2\bar{\Lambda}_f}{9} \left(-\frac{3}{2\bar{\Lambda}_f} \right) \right] = \frac{1}{1 + f'(\bar{R})}. \quad (33)$$

We emphasize that the residue is positive only for $1 + f'(\bar{R}) > 0$, indicating the case that the ghost is absent. In the limit of $\bar{\Lambda}_f \rightarrow 0$ ($f(R) \rightarrow 0$), the residue is 1, and thus, the amplitude describes a massless graviton with 2 DOF like

$$\lim_{\bar{\Lambda}_f \rightarrow 0} [4A] = 2 \left[\tilde{T}_{\mu\nu} \frac{1}{-\bar{\nabla}^2} T^{\mu\nu} - \frac{1}{2} \tilde{T} \frac{1}{-\bar{\nabla}^2} T \right]. \quad (34)$$

(c) Pole at $\bar{\nabla}^2 = \frac{1+f'(\bar{R})}{3f''(\bar{R})} - \frac{4\bar{\Lambda}_f}{3} \equiv m_f^2$

We have a newly massive scalar propagation unless $f'(\bar{R}) = 0$ and $f''(\bar{R}) = 0$, which shows the equivalence between $f(R)$ gravity and scalar-tensor theory on AdS_4 space. This pole was first pointed in Ref.[21]. Actually, the presence of this pole reflects that we are working with $f(R)$ gravities. This pole never appears in GR with cosmological constant. A similar pole appears also when including αR^2 [22, 23]. In this case, we may regard αR^2 as one of $f(R)$ forms. The residue at this massive physical pole takes the form

$$\frac{1}{3(1+f'(\bar{R}))}. \quad (35)$$

We note that this residue is positive definite only for $1+f'(\bar{R}) > 0$, showing the ghost-free pole. In the limit of $\bar{\Lambda}_f \rightarrow 0$ ($f(R) \rightarrow 0$), the residue is $1/3$, and thus, the amplitude reduces to

$$\lim_{\bar{\Lambda}_f \rightarrow 0} [4A] = 2 \left[\tilde{T}_{\mu\nu} \frac{1}{-\bar{\nabla}^2} T^{\mu\nu} - \frac{1}{3} \tilde{T} \frac{1}{-\bar{\nabla}^2} T \right], \quad (36)$$

which shows the van DVZ discontinuity clearly when comparing to (34).

As was mentioned in Ref.[22], our starting action (1) with external sources provides a massless graviton with 2 DOF for $f(R) = 0$ on AdS_4 space, while it provides a massless graviton with a massive scalar with $3(=2+1)$ DOF when choosing $f(R) = \alpha R^2$ on AdS_4 space.

In this work, we have shown that a massless graviton $h_{\mu\nu}^{TT}$ ($5 \rightarrow 2$ DOF, after imposing the source-conservation law appropriately) and a massive scalar ψ (1 DOF) propagate on AdS_4 space using arbitrary $f(R)$ gravity model with external sources. Consequently, we show that there is the vDVZ discontinuity in $f(R)$ gravity models because $R+f(R)$ means GR with an additional scalar. In the limit of $\bar{\Lambda}_f \rightarrow 0$ ($f(R) \rightarrow 0$), we did not recover the one-particle amplitude for a massless graviton, but we did recover the massless limit of one-particle amplitude for a massive graviton. Also, there is no apparent absence of the discontinuity since we did not introduce the Fierz-Pauli mass term on AdS_4 space.

In the constant curvature background of $f(R)$ gravities, the combination of $f(\bar{R})$ and $f'(\bar{R})$ determines the effective cosmological constant $\bar{\Lambda}_f$, while the combination of $f(\bar{R})$, $f'(\bar{R})$ and $f''(\bar{R})$ determines the mass squared m_f^2 of an additional scalar in the perturbation analysis. Hence, we have recovered general relativity.

Finally, we wish to mention that there was also physical non-equivalence between GR and $f(R)$ gravities on different considerations and purely classical arguments. This has been observed in cosmological viability of $f(R)$ gravity as an ideal fluid and its compatibility with a matter dominated phase [26].

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